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Towards holographic gravity dual of $\mathcal{N}=1$ superconformal Chern-Simons gauge theory

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Abstract: As we re-examine the known holographic $\mathcal{N}=1$ supersymmetric renormalization group flow in four dimensions, we describe the mass-deformed Bagger-Lambert theory or equivalently the mass-deformed $\mathrm{U}(2)\times\mathrm{U}(2)$ Chern-Simons gauge theory with level $k=1$ or 2, that has G_2 symmetry, by adding a mass term for one of the eight adjoint superfields. We obtain a detailed correspondence between the fields of AdS_4 supergravity and composite operators of the infrared field theory in three dimensions. The geometric superpotential from an eleven dimensional viewpoint is obtained for M2-brane probe analysis.

Keywords: AdS-CFT Correspondence, M-Theory.

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Contents

1. Introduction

When one reduces 11-dimensional supergravity theory to four-dimensional $\mathcal{N} = 8$ gauged supergravity, the four-dimensional spacetime is warped by warp factor that depends on both four-dimensional coordinates and 7-dimensional internal coordinates. This warp factor provides an understanding of the different relative scales of the 11-dimensional solutions corresponding to the critical points in $\mathcal{N} = 8$ gauged supergravity. An important aspect of the holographic duals [1] is the notion that the radial coordinate of AdS_4 can be viewed as a measure of energy. A supergravity kink description interpolating between $r \to \infty$ and $r \to -\infty$ can be interpreted as an explicit construction of the renormalization group(RG) flow between the ultraviolet(UV) fixed point and the infrared(IR) fixed point of the three dimensional boundary field theory.

It is known [2] that there exist five nontrivial critical points for the scalar potential of gauged $\mathcal{N} = 8$ supergravity: $\mathrm{SO}(7)^+, \mathrm{SO}(7)^-, G_2, \mathrm{SU}(4)^-$ and $\mathrm{SU}(3) \times \mathrm{U}(1)$. Among them G_2-invariant 7-ellipsoid and $\mathrm{SU}(3) \times \mathrm{U}(1)$-invariant stretched 7-ellipsoid are stable and supersymmetric. The holographic RG flow equations from $\mathcal{N} = 8$ SO(8)-invariant UV fixed point to $\mathcal{N} = 2$ $\mathrm{SU}(3) \times \mathrm{U}(1)$-invariant IR fixed point were constructed in [3]. Moreover, the holographic RG flow equations from $\mathcal{N} = 8$ SO(8)-invariant UV fixed point to $\mathcal{N} = 1$ G_2-invariant IR fixed point were obtained in [4, 5]. See also [6]. An exact solution to the 11-dimensional bosonic equations corresponding to the M-theory lift of the $\mathcal{N} = 2$ $\mathrm{SU}(3) \times \mathrm{U}(1)$-invariant RG flow was found in [7] and its Kahler structure was extensively studied in [8]. Furthermore, the M-theory lift of the $\mathcal{N} = 1$ G_2-invariant RG flow was done in [5].

Bagger and Lambert(BL) proposed a Lagrangian to describe the low energy dynamics of multiple M2-branes in [9]. See also related papers [10]. This BL theory is three dimensional $\mathcal{N} = 8$ supersymmetric theory with SO(8) global symmetry based on new 3-algebra and this 3-algebra with Lorentzian signature was proposed by [11]. The generators of the 3-algebra consist of the generators of an arbitrary semisimple Lie algebra and two additional null generators.

Very recently, in [12], in order to have arbitrary number of M2-branes, three dimensional Chern-Simons matter theories with gauge group $\mathrm{U}(N) \times \mathrm{U}(N)$ and level k which have $\mathcal{N} = 6$ superconformal symmetry are constructed. They describe this theory as the low energy limit of N M2-branes at $\mathbf{C}^4/\mathbf{Z}_k$ singularity. In particular, when $N = 2$, this leads to the BL theory. Furthermore, the full $\mathrm{SU}(4)_R$ symmetry of [12] is proved explicitly in [13]. By examining the holographic $\mathcal{N} = 2$ supersymmetric renormalization group flow solution among five nontrivial critical points above, in four dimensions, the mass-deformed BL theory that has $\mathrm{SU}(3)_I \times \mathrm{U}(1)_R$ symmetry is studied in [14] by the addition of mass term for one of the four adjoint chiral superfields. We list some relevant works on the M2-brane theory in [15]–[62].

Then it is natural to ask what happens for the holographic description with stable and $\mathcal{N} = 1$ supersymmetric G_2-invariant 7-ellipsoid compactification by deforming three dimensional Chern-Simons matter theories with gauge group $\mathrm{U}(N) \times \mathrm{U}(N)$ and level k? As a first step, we consider the $\mathrm{U}(2) \times \mathrm{U}(2)$ Chern-Simons gauge theory of [12] with level $k = 1$ or $k = 2$ which preserves G_2 global symmetry.

In this paper, starting from the first order differential equations, that are the supersymmetric flow solutions in four dimensional $\mathcal{N} = 8$ gauged supergravity interpolating between an exterior AdS_4 region with maximal $\mathcal{N} = 8$ supersymmetry and an interior AdS_4 with one eighth(i.e., $\mathcal{N} = 1$) of the maximal supersymmetry, we would like to interpret this as the RG flow in BL theory [1] which has $OSp(8|4)$ symmetry broken to the deformed BL theory which has $OSp(1|4)$ symmetry by the addition of a mass term for one of the eight adjoint superfields.

An exact correspondence is obtained between fields of bulk supergravity in the AdS_4 region in four dimensions and composite operators of the IR field theory in three dimensions. It is easy to check how the supersymmetry breaks for specific deformation and one can extract the correct full superpotential including the superpotential before the deformation also. The three dimensional analog of Leigh-Strassler RG flow in mass-deformed BL theory in three dimensions is expected by looking at its holographic dual theory in four dimensions along the line of [14].

In section 2, we review the supergravity solution in four dimensions in the context of RG flow, describe two supergravity critical points and present the supergravity multiplet in terms of G_2 invariant ones. The decomposition of SO(8) into G_2 is presented.

[1] When we describe BL theory in this paper, the two M2-branes theory($N = 2$) is equivalent to $\mathrm{U}(2) \times \mathrm{U}(2)$ Chern-Simons gauge theory of [12] with level $k = 1$ or $k = 2$.

In section 3, we deform BL theory by adding one of the mass term among eight superfields, along the lines of [63], write down the $\mathrm{SO}(7)^+$-invariant superpotential in $\mathcal{N}=1$ superfields which will be invariant under the G_2 after integrating out the massive superfield and describe the scale dimensions for the superfields at UV.

In section 4, the $OSp(1|4)$ representations(energy and spin) and G_2 representations in the supergravity mass spectrum for each multiplet at the $\mathcal{N}=1$ critical point and the corresponding $\mathcal{N}=1$ superfield in the boundary gauge theory are given. For this computation, the 11-dimensional "geometric" superpotential which reduces to the usual superpotential for the particular internal coordinate is needed to analyze the M2-brane analysis.

In section 5, we end up with the future directions.

2. The holographic $\mathcal{N}=1$ supersymmetric RG flow in four dimensions

In $\mathcal{N}=8$ supergravity [64], there exists an $\mathcal{N}=1$ supersymmetric G_2-invariant vacuum [65]. To arrive at this critical point, one should turn on expectation values of both scalar and pseudo-scalar fields where the completely antisymmetric self-dual and anti-self-dual tensors are invariant under G_2 since the G_2 is the common subgroup of $\mathrm{SO}(7)^+$ acting on the scalar and $\mathrm{SO}(7)^-$ acting on the pseudo-scalar respectively. Then the 56-bein can be written as 56×56 matrix whose elements are some functions of these scalar and pseudo-scalars. Then the G_2-invariant scalar potential of $\mathcal{N}=8$ supergravity in terms of the original variables of [65] is given by, through the superpotential found in [4, 5],

$$V(\lambda,\alpha) = g^2\left[\frac{16}{7}\left(\frac{\partial W}{\partial\lambda}\right)^2 + \frac{2}{7p^2q^2}\left(\frac{\partial W}{\partial\alpha}\right)^2 - 6\,W^2\right], \tag{2.1}$$

where g is a coupling of the theory, we introduce the hyperbolic functions of λ as

$$p=\cosh\left(\frac{\lambda}{2\sqrt{2}}\right), \qquad q=\sinh\left(\frac{\lambda}{2\sqrt{2}}\right) \tag{2.2}$$

and the superpotential, which can be read off from the element of A_1 tensor of the theory, is a magnitude of complex function on the variables λ and α

$$W(\lambda,\alpha) = |p^7 + e^{7i\alpha}q^7 + 7\left(p^3q^4e^{4i\alpha}+p^4q^3e^{3i\alpha}\right)|. \tag{2.3}$$

There exist two critical points and let us summarize these in table 1.

SO(8) critical point. This is well-known, trivial critical point at which the λ field vanishes with arbitrary α and whose cosmological constant $\Lambda=-6g^2$ from (2.1) and which preserves $\mathcal{N}=8$ supersymmetry.

G_2 critical point. There is a critical point at $\lambda=\sqrt{2}\sinh^{-1}\sqrt{\frac{2}{5}(\sqrt{3}-1)}$ and $\alpha=\cos^{-1}\frac{1}{2}\sqrt{3-\sqrt{3}}$ and the cosmological constant $\Lambda=-\frac{216\sqrt{2}}{25\sqrt{5}}\,3^{1/4}g^2$. This critical point has an unbroken $\mathcal{N}=1$ supersymmetry.

Symmetry	λ	α	V	W
SO(8)	0	any	$-6\,g^2$	1
G_2	$\sqrt{2}\,\sinh^{-1}\sqrt{\frac{2}{5}(\sqrt{3}-1)}$	$\cos^{-1}\frac{1}{2}\sqrt{3-\sqrt{3}}$	$-\frac{216\sqrt{2}}{25\sqrt{5}}\,3^{1/4}g^2$	$\sqrt{\frac{36\sqrt{2}\,3^{1/4}}{25\sqrt{5}}}$

Table 1: Summary of two critical points with symmetry group, supergravity fields(λ and α), scalar potential(V) and superpotential(W).

For the supergravity description of the nonconformal RG flow from one scale to another connecting these two critical points, the three dimensional Poincare invariant metric has the form $ds^2 = e^{2A(r)}\eta_{\mu'\nu'}dx^{\mu'}dx^{\nu'} + dr^2$ where $\eta_{\mu'\nu'} = (-,+,+)$ and r is the coordinate transverse to the domain wall. Then the supersymmetric flow equations [4, 5] with (2.3) and (2.2) are described as

$$\frac{d\lambda}{dr} = \frac{8\sqrt{2}}{7}\,g\,\partial_\lambda W, \qquad \frac{d\alpha}{dr} = \frac{\sqrt{2}}{7p^2q^2}\,g\,\partial_\alpha W, \qquad \frac{dA}{dr} = -\sqrt{2}\,g\,W. \tag{2.4}$$

We'll come back these flow equations when we discuss about the scalar function at IR in section 4.

The fields of the $\mathcal{N}=8$ theory transforming in SO(8) representations should be decomposed into G_2 representations. According to the decomposition SO(8) $\rightarrow G_2$ given in (A.3) which can be obtained from the branching rules of SO(8) $\rightarrow$ SO(7) by (A.1) and SO(7) $\rightarrow G_2$ by (A.2) we present in the appendix A, the spin $\frac{3}{2}$ field breaks into a singlet and a fundamental of G_2

$$\text{Spin } \frac{3}{2}: \qquad \mathbf{8} \rightarrow [\mathbf{1}] \oplus \mathbf{7} \tag{2.5}$$

under SO(8) $\rightarrow G_2$ and the singlet in square bracket corresponds to the massless graviton of the $\mathcal{N}=1$ theory. From the further branching rule of $G_2 \rightarrow$ SU(3) by (A.4) one sees that the above septet($\mathbf{7}$) which will be located at $\mathcal{N}=1$ massive gravitino multiplet breaks into triplet($\mathbf{3}$) and anti-triplet($\overline{\mathbf{3}}$) that enter into the component of $\mathcal{N}=2$ short massive gravitino multiplet as well as a singlet($\mathbf{1}$) which enters into the component of $\mathcal{N}=2$ massless graviton multiplet under the $G_2 \rightarrow$ SU(3) breaking. Although the global symmetry G_2 is reduced to SU(3), the $\mathcal{N}=1$ supersymmetry is enhanced to $\mathcal{N}=2$ supersymmetry.

According to the decomposition SO(8) $\rightarrow G_2$ given in (A.3), one obtains the following decomposition by a singlet, two fundamentals, adjoint and a symmetric representation

$$\text{Spin } \frac{1}{2}: \qquad \mathbf{56} \rightarrow \mathbf{1} \oplus [\mathbf{7}] \oplus \mathbf{7} \oplus [\mathbf{14}] \oplus \mathbf{27}, \tag{2.6}$$

and the seven Goldstino modes that are absorbed into massive spin $\frac{3}{2}$ fields (2.5) are identified with septet in square bracket and the adjoint representation fourteen($\mathbf{14}$) in square bracket corresponds to the massless vector multiplet of the $\mathcal{N}=1$ theory. From the further branching rule of $G_2 \rightarrow$ SU(3) by (A.4) one sees that the above twenty-seven representation($\mathbf{27}$) which will be located at Wess-Zumino multiplet breaks into a

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singlet($\mathbf{1}$)(entering into the component of $\mathcal{N}=2$ long massive vector multiplet), triplet($\mathbf{3}$) and anti-triplet($\overline{\mathbf{3}}$)(entering into the component of $\mathcal{N}=2$ short massive gravitino multiplet), sextet($\mathbf{6}$) and anti-sextet($\overline{\mathbf{6}}$)(entering into the component of $\mathcal{N}=2$ short massive hypermultiplet) and octet($\mathbf{8}$)(entering into the component of $\mathcal{N}=2$ massless vector multiplet) under the $G_2 \to \mathrm{SU}(3)$ breaking. Moreover, a singlet in (2.6) which will be located at Wess-Zumino multiplet goes to the component of $\mathcal{N}=2$ long massive vector multiplet while the septet in (2.6) which will be located at massive gravitino multiplet goes to the component of $\mathcal{N}=2$ long massive vector multiplet and the component of $\mathcal{N}=2$ short massive gravitino multiplet.

For spin 1 field, one has the folllowing breaking under $\mathrm{SO}(8) \to G_2$ leading to two fundamentals and adjoint

$$\text{Spin 1}: \qquad \mathbf{28} \to \mathbf{7} \oplus \mathbf{7} \oplus [\mathbf{14}] \tag{2.7}$$

which implies that the representation fourteen($\mathbf{14}$) in square bracket corresponds to the massless vector multiplet of the $\mathcal{N}=1$ theory. The two septets which will be located at massive gravitino multiplet break into two triplets and anti-triplets, that enter into the component of $\mathcal{N}=2$ short massive gravitino multiplet, a singlet which enters into the component of $\mathcal{N}=2$ massless graviton multiplet and a singlet that enters into the component of $\mathcal{N}=2$ long massive vector multiplet under the further $G_2 \to \mathrm{SU}(3)$ breaking.

For spin zero field, the breaking goes to a sum of two singlets, two fundamentals and two symmetric representations

$$\text{Spin 0}: \qquad \mathbf{70} \to \mathbf{1} \oplus \mathbf{1} \oplus [\mathbf{7} \oplus \mathbf{7}] \oplus \mathbf{27} \oplus \mathbf{27} \tag{2.8}$$

under $\mathrm{SO}(8) \to G_2$ and the fourteen Goldstone bosons modes are identified with two septets in square bracket. Their quantum numbers are in agreement with those of massive vectors in (2.7). The two twenty-seven representations which will be located at Wess-Zumino multiplet break into two singlets(entering into the component of $\mathcal{N}=2$ long massive vector multiplet), triplet and anti-triplet(entering into the component of $\mathcal{N}=2$ short massive gravitino multiplet), two sextets and two anti-sextets(entering into the component of $\mathcal{N}=2$ short massive hypermultiplet) and two octets(entering into the component of $\mathcal{N}=2$ massless vector multiplet) under the further $G_2 \to \mathrm{SU}(3)$ breaking. The remaining triplet and anti-triplet can be identified with six Goldstone boson modes under the further $G_2 \to \mathrm{SU}(3)$ breaking. Moreover, two singlets in (2.8) which will be located at Wess-Zumino multiplet enter into the component of $\mathcal{N}=2$ long massive vector multiplet.

Finally, from the breaking

$$\text{Spin 2}: \qquad \mathbf{1} \to [\mathbf{1}] \tag{2.9}$$

under $\mathrm{SO}(8) \to G_2$, this field is located at $\mathcal{N}=1$ massless graviton multiplet. Of course, under the $\mathrm{SU}(3)$, this enters into the component of $\mathcal{N}=2$ massless graviton multiplet.

We'll reorganize (2.5), (2.6), (2.7), (2.8) and (2.9) in the context of supergravity multiplet with corresponding $OSp(1|4)$ quantum numbers in section 4. The singlets are placed

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at Wess-Zumino multiplet and massless graviton multiplet, septets are located at massive gravitino multiplet, the adjoints are at massless vector multiplet and twenty seven representations sit in another Wess-Zumino multiplet.

3. An $\mathcal{N}=1$ supersymmetric membrane flow in three dimensional deformed theory

Let us describe the deformed BL theory by adding eight mass parameters $m_1, \cdots, m_8$. The theory by [12] is closely related to the BL theory. Moreover, it is conjectured in [13] that the $\mathrm{SU}(3)_I \times \mathrm{U}(1)_Y$-invariant $\mathrm{U}(2) \times \mathrm{U}(2)$ Chern-Simons gauge theory with level $k=1$ with the effective 6-th order superpotential is dual to the background by two units of four-form flux while the theory at level $k=2$ is dual to a $\mathbf{Z}_2$ orbifold of the background in [7, 66]. Recall that the self-dual and anti self-dual tensors that are invariant under the G_2 symmetry in $\mathcal{N}=8$ gauged supergravity are given by [2, 6, 3, 4, 67, 5, 66, 68]

$$\begin{aligned} Y^+_{ijkl} &= [(\delta^{1234}_{ijkl} + \delta^{5678}_{ijkl}) + (\delta^{1256}_{ijkl} + \delta^{3478}_{ijkl}) + (\delta^{3456}_{ijkl} + \delta^{1278}_{ijkl})] \\ &\quad + [-(\delta^{1357}_{ijkl} + \delta^{2468}_{ijkl}) + (\delta^{2457}_{ijkl} + \delta^{1368}_{ijkl}) + (\delta^{2367}_{ijkl} + \delta^{1458}_{ijkl}) + (\delta^{1467}_{ijkl} + \delta^{2358}_{ijkl})], \\ Y^-_{ijkl} &= i[(\delta^{1234}_{ijkl} - \delta^{5678}_{ijkl}) + (\delta^{1256}_{ijkl} - \delta^{3478}_{ijkl}) + (\delta^{3456}_{ijkl} - \delta^{1278}_{ijkl})] \\ &\quad + i[-(\delta^{1357}_{ijkl} - \delta^{2468}_{ijkl}) + (\delta^{2457}_{ijkl} - \delta^{1368}_{ijkl}) + (\delta^{2367}_{ijkl} - \delta^{1458}_{ijkl}) + (\delta^{1467}_{ijkl} - \delta^{2358}_{ijkl})]. \end{aligned} \tag{3.1}$$

Turning on the scalar fields proportional to Y^+_{ijkl} yields an $\mathrm{SO}(7)^+$ invariant vacuum while turning on the pseudo-scalar fields proportional to Y^-_{ijkl} yields an $\mathrm{SO}(7)^-$ invariant vacuum. Simultaneous turning on both scalar and pseudo-scalar fields leads to G_2-invariant vacuum with $\mathcal{N}=1$ supersymmetry. The choice of the mass parameters we describe corresponds to the self-dual tensor or anti self-dual tensor for the indices 5678, 3478, 3456, 2468, 2457, 2367, 2358, if we shift all the indices by adding 2 to (3.1), besides an identity. In [63] there are three mass parameters while in [14] there exist four mass parameters. Then the fermionic mass terms from [9] are given by

$$\begin{aligned} \mathcal{L}_{f.m.} &= -\frac{i}{2} h_{ab} \bar{\Psi}^a \left(m_1 \Gamma^{78910} + m_2 \Gamma^{56910} - m_3 \Gamma^{5678} + m_4 \Gamma^{46810} \right. \\ &\qquad \left. + m_5 \Gamma^{4679} + m_6 \Gamma^{4589} - m_7 \Gamma^{45710} - m_8 \mathbf{1} \right) \Psi^b. \end{aligned} \tag{3.2}$$

The indices $a, b, \cdots$ run over the adjoint of the Lie algebra. Then the corresponding fermionic supersymmetric transformation is given by

$$\begin{aligned} \delta_m \Psi^a &= \left(m_1 \Gamma^{78910} + m_2 \Gamma^{56910} - m_3 \Gamma^{5678} + m_4 \Gamma^{46810} \right. \\ &\qquad \left. + m_5 \Gamma^{4679} + m_6 \Gamma^{4589} - m_7 \Gamma^{45710} - m_8 \mathbf{1} \right) X^a_I \Gamma_I \epsilon. \end{aligned} \tag{3.3}$$

We impose the constraints on the ϵ parameter that satisfies the $\frac{1}{8}$ BPS condition(the number of supersymmetries is two):

$$\begin{aligned} \Gamma^{5678}\epsilon &= \Gamma^{56910}\epsilon = \Gamma^{78910}\epsilon = \Gamma^{46810}\epsilon = \Gamma^{34910}\epsilon = \Gamma^{3478}\epsilon = \Gamma^{3456}\epsilon \\ &= -\Gamma^{4679}\epsilon = -\Gamma^{4589}\epsilon = -\Gamma^{45710}\epsilon = -\epsilon. \end{aligned} \tag{3.4}$$

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The first three conditions in (3.4) provide $\frac{1}{4}$ BPS condition and the remaining seven conditions restrict to the ϵ parameter further and we are left with the right number of supersymmetry we are dealing with.

Let us introduce the bosonic mass term which preserves $\mathcal{N}=1$ supersymmetry:

$$\mathcal{L}_{b.m.} = -\frac{1}{2} h_{ab} X_I^a (m^2)_{IJ} X_J^b. \tag{3.5}$$

Using the supersymmetry variation for X_I^a, $\delta X_I^a = i\bar{\epsilon}\Gamma_I \Psi^a$, and the supersymmetry variation for Ψ^a by the equation (3.3), the variation for the bosonic mass term (3.5) plus the fermionic mass term (3.2) leads to

$$\begin{aligned} \delta\mathcal{L} = & \; i h_{ab} X_I^a (m^2)_{IJ} \bar{\Psi}^b \Gamma_J \epsilon \\ & - i h_{ab} \bar{\Psi}^a \left(m_1 \Gamma^{78910} + m_2 \Gamma^{56910} - m_3 \Gamma^{5678} + m_4 \Gamma^{46810} \right. \\ & \left. + m_5 \Gamma^{4679} + m_6 \Gamma^{4589} - m_7 \Gamma^{45710} - m_8 \mathbf{1} \right)^2 X_I^b \Gamma_I \epsilon. \end{aligned} \tag{3.6}$$

In order to vanish this, the bosonic mass term $(m^2)_{IJ}\Gamma_J$ should take the form

$$\begin{aligned} (m^2)_{IJ}\Gamma^J \rightarrow & \; (m_1+m_2-m_3+m_4-m_5-m_6+m_7-m_8)^2\Gamma_3 \\ & + (m_1+m_2-m_3-m_4+m_5+m_6-m_7-m_8)^2\Gamma_4 \\ & + (m_1-m_2+m_3+m_4-m_5+m_6-m_7-m_8)^2\Gamma_5 \\ & + (m_1-m_2+m_3-m_4+m_5-m_6+m_7-m_8)^2\Gamma_6 \\ & + (m_1-m_2-m_3-m_4-m_5+m_6+m_7+m_8)^2\Gamma_7 \\ & + (m_1-m_2-m_3+m_4+m_5-m_6-m_7+m_8)^2\Gamma_8 \\ & + (m_1+m_2+m_3-m_4-m_5-m_6-m_7+m_8)^2\Gamma_9 \\ & + (m_1+m_2+m_3+m_4+m_5+m_6+m_7+m_8)^2\Gamma_{10} \end{aligned} \tag{3.7}$$

by computing the mass terms for second and third lines of (3.6) explicitly.[2] When all the mass parameters are equal

$$m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m,$$

then the diagonal bosonic mass term in (3.7) has nonzero component only for 1010 and other components(33, 44, 55, 66, 77, 88 and 99) are vanishing. This resembles the structure of A_1^{IJ} tensor of AdS_4 supergravity where the A_1^{IJ} tensor has two distinct eigenvalues with degeneracies 7 and 1 respectively [65, 5]. The degeneracy 1 is related to the $\mathcal{N}=1$ supersymmetry. Then one obtains the bosonic mass term which appears in (3.5)

$$(m^2)_{IJ} = \mathrm{diag}(0,0,0,0,0,0,0,64m^2). \tag{3.8}$$

[2] The relevant terms become $m_1^2+m_2^2+m_3^2+m_4^2+m_5^2+m_6^2+m_7^2+m_8^2+2(-m_1m_2-m_4m_7-m_5m_6+m_3m_8)\Gamma^{5678}+2(m_1m_3+m_4m_6+m_5m_7-m_2m_8)\Gamma^{56910}+2(m_1m_4+m_2m_7-m_3m_6-m_5m_8)\Gamma^{4679}+2(m_1m_5+m_2m_6+m_3m_7-m_4m_8)\Gamma^{46810}+2(-m_1m_6-m_2m_5-m_3m_4+m_7m_8)\Gamma^{45710}+2(m_1m_7+m_2m_4+m_3m_5-m_6m_8)\Gamma^{4589}+2(m_2m_3+m_4m_5+m_6m_7-m_1m_8)\Gamma^{78910}$ explicitly. We also used the conditions (3.4).

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Let us introduce the eight $\mathcal{N}=1$ superfields as follows:

$$\begin{aligned}
&\Phi_1 = X_3 + \cdots , && \Phi_2 = X_4 + \cdots , && \Phi_3 = X_5 + \cdots , \\
&\Phi_4 = X_6 + \cdots , && \Phi_5 = X_7 + \cdots , && \Phi_6 = X_8 + \cdots , \\
&\Phi_7 = X_9 + \cdots , && \Phi_8 = X_{10} + \cdots , &&
\end{aligned} \tag{3.9}$$

where we do not include the $\mathcal{N}=1$ fermionic fields. The $\Phi_1, \cdots, \Phi_7$ constitute a fundamental **7** representation of G_2 while Φ_8 is a singlet **1** of G_2. The potential in the BL theory [9] is given by

$$\frac{1}{3\kappa^2} h_{ab} f_{cde}{}^{a} X_I^c X_J^d X_K^e f_{fgh}{}^{b} X_I^f X_J^g X_K^h$$

where κ is a Chern-Simons coefficient. In terms of $\mathcal{N}=1$ superfields, this contains the following expressions

$$\frac{1}{\kappa^2} h_{ab} f_{cde}{}^{a} f_{fgh}{}^{b} \left(\Phi_1^c \Phi_2^d \Phi_3^e \Phi_1^f \Phi_2^g \Phi_3^h + \text{other terms} \right)$$

by using the relation (3.9) between the component fields and superfields. This provides the superpotential and it is given by [69] with (3.1)

$$\frac{1}{\kappa} f_{abcd} Y^{+ijkl} \, \mathrm{Tr}\, \Phi_i^a \Phi_j^b \Phi_k^c \Phi_l^d. \tag{3.10}$$

This form has manifest $\mathrm{SO}(7)^+$ global symmetry and by fixing the coefficients of $\mathcal{N}=1$ superspace action [69] this global symmetry is enhanced to SO(8) symmetry with maximal supersymmetry. In $\mathcal{N}=1$ language, the superpotential consisting of the mass term (3.8) and quartic term (3.10) where we absorbed the κ into the structure constant is given by

$$\begin{aligned}
W = \frac{1}{2} M \, \mathrm{Tr}\, \Phi_8^2 &+ (\mathrm{Tr}\, \Phi_1\Phi_2\Phi_7\Phi_8 + \text{other 6-terms with } \Phi_8) \\
&+ (\mathrm{Tr}\, \Phi_3\Phi_4\Phi_5\Phi_6 + \text{other 6-terms without } \Phi_8) \, .
\end{aligned}$$

The fourteen terms except the first term are the superpotential required by $\mathcal{N}=8$ supersymmetry and the first term breaks $\mathcal{N}=8$ down to $\mathcal{N}=1$. The theory has matter multiplet in seven flavors $\Phi_1, \Phi_2, \cdots, \Phi_7$ transforming in the adjoint with flavor symmetry under which the matter multiplet forms a septet(**7**) of the $\mathcal{N}=1$ theory. Therefore, we turn on the mass perturbation in the UV and flow to the IR. This maps to turning on certain fields in the AdS_4 supergravity where they approach to zero in the UV($r \to \infty$) and develop a nontrivial profile as a function of r as one goes to the IR($r \to -\infty$). We can integrate out the massive scalar Φ_8 with adjoint index at a low enough scale and this results in the 6-th order superpotential $\mathrm{Tr}(Y^{+ijk8}\Phi_i\Phi_j\Phi_k)^2 + \mathrm{Tr}\, \epsilon_{ijklmnp} Y^{+ijk8}(\Phi_l\Phi_m\Phi_n\Phi_p)$. The scale dimensions of eight superfields $\Phi_i (i = 1, 2, \cdots, 8)$ are $\Delta_i = \frac{1}{4}$ at the UV. This is because the sum of Δ_i is equal to the canonical dimension of the superpotential which is $3 - 1 = 2$ [70]. By symmetry, one arrives at $\Delta_i = \frac{1}{4}$.

Thus we have found $\mathcal{N}=1$ superconformal Chern-Simons theories with global G_2 symmetry and $k=1$ Chern-Simons gauge theories with G_2-invariant superpotential deformation are dual to the holographic RG flows in [5]. We expect that G_2-invariant $\mathrm{U}(N) \times \mathrm{U}(N)$

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Boundary Operator	Energy	Spin 0	Spin $\frac{1}{2}$
$S = \left(\sum_{i=1}^{7} \Phi_i^2\right)^{\frac{6}{5}}$	E_0	**1**	
	$E_0 + \frac{1}{2}$		**1**
	$E_0 + 1$	**1**	

Table 2: The $OSp(1|4)$ representations(energy, spin) and G_2 representations singlets in the supergravity mass spectrum for Wess-Zumino multiplet at the $\mathcal{N} = 1$ critical point and the corresponding $\mathcal{N} = 1$ superfield S in the boundary gauge theory.

Chern-Simons gauge theory for $N > 2$ with $k = 1, 2$ where there exists an enhancement of $\mathcal{N} = 8$ supersymmetry [12, 13] is dual to the background of [5] with N unit of flux. In next section, the gauge invariant composites in the superconformal field theory at the IR in three dimensions are mapped to the corresponding supergravity bulk fields in four dimensions.

4. The $OSp(1|4)$ spectrum and operator map between bulk and boundary theories

A further detailed correspondence between fields of AdS_4 supergravity in four dimensions and composite operators of the IR field theory in three dimensions is described in this section.

The even subalgebra of the superalgebra $OSp(1|4)$ is $Sp(4, R) \simeq \mathrm{SO}(3, 2)$ that is the isometry algebra of AdS_4 [71]. The maximally compact subalgebra is then $\mathrm{SO}(2)_E \times \mathrm{SO}(3)_S$ where the generator of $\mathrm{SO}(2)_E$ is the hamiltonian of the system and its eigenvalues E are the energy levels of states for the system, the group $\mathrm{SO}(3)_S$ is the rotation group and its representation s describes the spin states of the system. A supermultiplet, a unitary irreducible representations(UIR) of the superalgebra $OSp(1|4)$, consists of a finite number of UIR of the even subalgebra and a particle state is characterized by a spin s and energy E.

Let us classify the supergravity multiplet, which is invariant under G_2, we explained in section 2 and describe them in the three dimensional boundary theory.

Wess-Zumino multiplet. The conformal dimension Δ, which is irrational and unprotected, is given by $\Delta = E_0 > \frac{1}{2}$. Let us denote $S(x, \theta)$, that is a scalar superfield, by the corresponding boundary operator in boundary gauge theory. This scalar field has a dimension $\frac{5}{6}(6 + \sqrt{3})$ in the IR. We'll come back this issue at the end of this section. The corresponding $OSp(1|4)$ representations and corresponding $\mathcal{N} = 1$ superfield in three dimensions are listed in table 2.

Wess-Zumino multiplet. The conformal dimension Δ for the lowest component of this multiplet is given by $\Delta = E_0 > \frac{1}{2}$. The AdS_4 supergravity multiplet corresponds to the scalar superfield $\Phi(x, \theta)$. That is, in the θ expansion, there are three component fields in the bulk. Then the bilinear of the seven Φ_i superfields by symmetrizing the two G_2

Boundary Operator	Energy	Spin 0	Spin $\frac{1}{2}$
$\mathrm{Tr}\,\Phi_{(i}\Phi_{j)}$	E_0	**27**	
	$E_0 + \frac{1}{2}$		**27**
	$E_0 + 1$	**27**	

Table 3: The $OSp(1|4)$ representations(energy, spin) and G_2 symmetric representations in the supergravity mass spectrum for Wess-Zumino multiplet at the $\mathcal{N} = 1$ critical point and the corresponding $\mathcal{N} = 1$ superfield in the boundary gauge theory.

Boundary Operator	Energy	Spin $\frac{1}{2}$	Spin 1	Spin $\frac{3}{2}$
$\mathrm{Tr}\,D_\alpha W_\beta \Phi_j$	E_0		**7**	
	$E_0 + \frac{1}{2}$	**7**		**7**
	$E_0 + 1$		**7**	

Table 4: The $OSp(1|4)$ representations(energy, spin) and G_2 fundamental representations in the supergravity mass spectrum for massive gravitino multiplet at the $\mathcal{N} = 1$ critical point and the corresponding $\mathcal{N} = 1$ superfield in the boundary gauge theory.

indices present in them provides a symmetric representation of G_2, **27**, corresponding to $\mathrm{Tr}\,\Phi_{(i}\Phi_{j)}$. Note that from the tensor product between two **7**'s of SO(7), one writes down $\mathbf{7} \times \mathbf{7} = \mathbf{1}_s \oplus \mathbf{21}_a \oplus \mathbf{27}_s$. Using the branching rule of (A.2), this leads to $\mathbf{1} \oplus \mathbf{7} \oplus \mathbf{14} \oplus \mathbf{27}_s$ under the breaking SO(7) into G_2. Therefore, one obtains the symmetric representation **27** of G_2. Group theoretically, the structure of this Wess-Zumino multiplet and $\mathcal{N} = 2$ short massive hypermultiplet in [14] looks similar. The symmetric representations **6** and $\bar{\mathbf{6}}$ of SU(3) originate from only this symmetric representation **27** of G_2 when we look at the branching rule by (A.4). As we observed in section 2, through the supersymmetry enhancement from $\mathcal{N} = 1$ to $\mathcal{N} = 2$, some other components among the repesentation **27** of G_2 distribute into other $\mathcal{N} = 2$ multiplets while the symmetric representations **6** and $\bar{\mathbf{6}}$ of SU(3) remain and constitute $\mathcal{N} = 2$ short massive hypermultiplet. The corresponding $OSp(1|4)$ representations and corresponding superfield are listed in table 3.

Massive gravitino multiplet. The conformal dimension Δ is given by $\Delta = E_0 > 2$. This corresponds to spinorial superfield $\Phi_{\alpha\beta}(x, \theta)$. In the θ expansion, the component fields in the bulk are located with appropriate quantum numbers. Then one can identify $\mathrm{Tr}\,D_\alpha W_\beta \Phi_j$ with **7** of G_2. Although the group structure between this massive gravitino multiplet and $\mathcal{N} = 2$ short massive gravitino multiplet is different from each other, they have both fundamental representations and one obtains this multiplet when one takes $\mathcal{N} = 1$ superderivative D_α on $\mathcal{N} = 2$ short massive gravitino multiplet in [14]. Some of the fundamental and anti-fundamental representations **3** and $\bar{\mathbf{3}}$ of SU(3) in $\mathcal{N} = 2$ theory originate from this fundamental representation **7** of G_2 when we look at the branching rule by (A.4) and some of them come from others **14** and **27** of G_2. As we observed in section 2, from $\mathcal{N} = 1$ to $\mathcal{N} = 2$, the singlets among the representation **7** of G_2 enter into $\mathcal{N} = 2$ massless graviton multiplet or long massive vector multiplet. The corresponding $OSp(1|4)$ representations and corresponding superfield are listed in table 4.

Boundary Operator	Energy	Spin $\frac{1}{2}$	Spin 1	Spin $\frac{3}{2}$	Spin 2
$\mathrm{Tr}\, D_\alpha \Phi T^A \Phi$	$E_0 = \frac{3}{2}$	$[\mathbf{14}]$			
	$E_0 + \frac{1}{2} = 2$		$[\mathbf{14}]$		
$D^\alpha T^{\beta\gamma}$	$E_0 = \frac{5}{2}$			$[\mathbf{1}]$	
	$E_0 + \frac{1}{2} = 3$				$[\mathbf{1}]$

Table 5: The $OSp(1|4)$ representations(energy, spin) and G_2 representations(adjoints and singlets) in the supergravity mass spectrum for massless vector and graviton multiplets at the $\mathcal{N}=1$ critical point and the corresponding $\mathcal{N}=1$ superfields in the boundary gauge theory.

$\mathcal{N}=1$ massless graviton multiplet. The bulk field $\Phi_{\alpha\beta\gamma}(x,\theta)$ can be identified with $D^\alpha T^{\beta\gamma}(x,\theta)$ where $T^{\beta\gamma}(x,\theta)$ is the stress energy tensor superfield. In components, the θ expansion of this superfield has the $\mathcal{N}=1$ supercurrent and the stress energy tensor. Also in this case, one obtains this multiplet when one takes $\mathcal{N}=1$ superderivative D^α on $\mathcal{N}=2$ massless graviton multiplet in [14]. The conformal dimension $\Delta = \frac{5}{2}$. The structure of this massless graviton multiplet and $\mathcal{N}=2$ massless graviton multiplet looks similar to each other. Some of the singlet representation $\mathbf{1}$ of SU(3) in $\mathcal{N}=2$ theory originate from this singlet representation $\mathbf{1}$ of G_2 when we look at the branching rule by (A.4) and some of them come from $\mathbf{7}$ and $\mathbf{27}$ of G_2. The corresponding $OSp(1|4)$ representations and corresponding superfield are listed in table 5.

$\mathcal{N}=1$ massless vector multiplet. This conserved vector current is given by a scalar superfield $D_\alpha J^A(x,\theta)$. This transforms in the adjoint representation of G_2 flavor group. The boundary object is given by $\mathrm{Tr}\, D_\alpha \Phi T^A \Phi$ where the generator T^A is $N \times N$ matrix with $A = 1, 2, \cdots, N^2 - 1$ for general N. One obtains also this multiplet when one takes $\mathcal{N}=1$ superderivative D_α on $\mathcal{N}=2$ massless vector multiplet in [14] although the group structure is different from each other but they have both adjoint representations. The conformal dimension $\Delta = \frac{3}{2}$. By taking a tensor product between two $\mathbf{7}$'s, one gets this adjoint $\mathbf{14}$ of G_2 representation as in Wess-Zumino multiplet above. The structure of this massless vector multiplet and $\mathcal{N}=2$ massless vector multiplet resembles each other. Some of the adjoint representation $\mathbf{8}$ of SU(3) in $\mathcal{N}=2$ theory originate from this adjoint representation $\mathbf{14}$ of G_2 when we look at the branching rule by (A.4) and some of them come from $\mathbf{27}$ of G_2. As we observed in section 2, from $\mathcal{N}=1$ to $\mathcal{N}=2$, triplets and anti-triplets among the representation $\mathbf{14}$ of G_2 enter into $\mathcal{N}=2$ short massive gravitino multiplet. The corresponding $OSp(1|4)$ representations and corresponding superfield are listed in table 5 also.

The 11-dimensional metric with warped product ansatz is given by [65, 72, 5, 66]

$$ds_{11}^2 = ds_4^2 + ds_7^2 = \Delta(x,y)^{-1}\, g_{\mu\nu}(x)\, dx^\mu dx^\nu + G_{mn}(x,y)\, dy^m dy^n,$$

where $\mu, \nu = 1, 2, \cdots, 4$ and $m, n = 1, 2, \cdots, 7$. The 4-dimensional metric which has a 3-dimensional Poincare invariance takes the form $g_{\mu\nu}(x)\, dx^\mu dx^\nu = e^{2A(r)}\, \eta_{\mu'\nu'}\, dx^{\mu'} dx^{\nu'} + dr^2$, where $\eta_{\mu'\nu'} = (-,+,+)$ and $r = x^4$ is the coordinate transverse to the domain wall as in section 2 and the scale factor $A(r)$ behaves linearly in r at UV and IR regions. The metric formula by [65] generates the 7-dimensional metric from the two input data of AdS_4 vacuum expectation values for scalar and pseudo-scalar fields (λ, α). Let us introduce the redefined fields [5]

$$a \equiv \cosh\left(\frac{\lambda}{\sqrt{2}}\right) + \cos\alpha \, \sinh\left(\frac{\lambda}{\sqrt{2}}\right), \qquad b \equiv \cosh\left(\frac{\lambda}{\sqrt{2}}\right) - \cos\alpha \, \sinh\left(\frac{\lambda}{\sqrt{2}}\right).$$

We recall that the two input data of (a, b) are

$$a = 1, \qquad b = 1 \tag{4.1}$$

for the SO(8)-invariant UV critical point whereas

$$a = \sqrt{\frac{6\sqrt{3}}{5}}, \qquad b = \sqrt{\frac{2\sqrt{3}}{5}} \tag{4.2}$$

for the G_2-invariant IR critical point.[3]

From the standard metric of a 7-dimensional ellipsoid, the diagonal 8×8 matrix Q_{AB} is given by $Q_{AB} = \mathrm{diag}\left(b^2, b^2, b^2, b^2, b^2, b^2, b^2, a^2\right)$ [5, 66] and the 7-dimensional ellipsoidal metric $ds^2_{EL(7)} = dX^A Q^{-1}_{AB} dX^B$ where X^A is a coordinate for R^8 and $A, B = 1, 2, \cdots, 8$ arises via

$$ds_7^2 = G_{mn}(x, y)\, dy^m dy^n = \sqrt{\Delta\, a}\, L^2 \left(a^{-2}\, \xi^2\, d\theta^2 + \sin^2\theta \, d\Omega_6^2 \right) \tag{4.3}$$

where the quadratic form ξ^2 is given by [65, 5, 66]

$$\xi^2 = a^2 \cos^2\theta + b^2 \sin^2\theta \tag{4.4}$$

and the warped factor Δ is given by

$$\Delta = a^{-1}\, \xi^{-\frac{4}{3}}. \tag{4.5}$$

The metric $d\Omega_6^2$ on $\mathbf{S}^6 \simeq G_2/\,\mathrm{SU}(3)$ in (4.3) preserves the Fubini-Study metric on $\mathbf{CP}^2$ [73, 66]. Note that the corresponding base 6-sphere for $\mathrm{SU}(3)_I \times \mathrm{U}(1)_R$-invariant sector [74, 14] is given by $\mathbf{CP}^3$ which is the homogeneous space $\mathrm{SU}(4)/[\mathrm{SU}(3) \times \mathrm{U}(1)]$ characterized by the Kahler form J [75, 66].

As in [7], let us go to the $\mathrm{SL}(8, R)$ basis and introduce the rotated vielbeins

$$\begin{aligned} U^{ij}{}_{IJ} &= u^{ij}{}_{ab} (\Gamma_{IJ})^{ab}, \qquad & V^{ijIJ} &= v^{ijab} (\Gamma_{IJ})^{ab} \\ U_{ij}{}^{IJ} &= u_{ij}^{ab} (\Gamma_{IJ})^{ab}, \qquad & V_{ijIJ} &= v_{ijab} (\Gamma_{IJ})^{ab} \end{aligned}$$

where 28-beins and gamma matrices are the same as those in [4]. Now let us define

$$\begin{aligned} A_{ijIJ} &= \frac{1}{\sqrt{2}} \left(U_{ij}{}^{IJ} + V_{ijIJ} \right), \qquad & B_{ij}^{\ IJ} &= \frac{1}{\sqrt{2}} \left(U_{ij}{}^{IJ} - V_{ijIJ} \right), \\ C^{ij}{}_{IJ} &= \frac{1}{\sqrt{2}} \left(U^{ij}{}_{IJ} + V^{ijIJ} \right), \qquad & D^{ijIJ} &= \frac{1}{\sqrt{2}} \left(-U^{ij}{}_{IJ} + V^{ijIJ} \right). \end{aligned}$$

[3] The scalar potential (2.1) can be rewritten as $V(a, b) = \frac{1}{8} a^2 \left(a^5 - 28 a^2 b + 14 a^3 b^2 - 84 b^3 + 49 a b^4 \right)$.

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Then "geometric" T-tensor can be written as

$$\widetilde{T}_l^{\,kij} = \frac{1}{168\sqrt{2}} C^{ij}_{\;\;LM} \left(A_{lmJK} D^{kmKI} \, \delta^L_I \, x_M x_J - B_{lm}^{\;\;\;JK} C^{km}_{\;\;\;KI} \, \delta^M_J \, x_L x_I \right) \tag{4.6}$$

and furthermore the "geometric" A_1-tensor is given by

$$\widetilde{A}_1^{ij} = \widetilde{T}_m^{\,imj}. \tag{4.7}$$

The idea of [76] is to replace δ^{IJ} in the original T-tensor with $x^I x^J$ but δ^I_J remains unchanged, as in (4.6).

Now the 88 component of $\widetilde{A}_1^{ij}$, $\widetilde{A}_1^{88}$, provides the "geometric" superpotential in terms of a, b and θ and from the complete expressions in appendix B (B.1), one gets

$$W_{gs} \equiv |\widetilde{A}_1^{88}|^2 = a^{\frac{3}{2}} \sqrt{(a^2 \cos^2\theta + b^2 \sin^2\theta)^2 - 16(ab-1)\sin^2\theta\cos^2\theta}. \tag{4.8}$$

This superpotential is different from W_{AI} in [5], in general. More explicitly, one obtains[4]

$$W_{gs}^2 = 4W_{AI}^2 + \frac{1}{4W^2} a^6 \left(-2+ab\right)^2 \left(-1+ab\right) \left(3+4\cos 2\theta\right)^2.$$

In particular, when $\theta = \cos^{-1}\frac{1}{\sqrt{8}}$, one obtains $W_{gs} = W = W_{AI}$. Although there are two different solutions for the superpotential, W_{gs} and W_{AI}, in 11-dimensions, there exists the same superpotential W in 4-dimensions. For SO(8) maximal $\mathcal{N}=8$ supersymmetric critical point with (4.1), it is easy to check $W_{gs} = 2W_{AI} = W = 1$.

Performing the M2-brane probe analysis [77, 76, 8], one can compute the effective Lagrangian for the probe moving at a small velocity transverse to its world-volume. If the potential vanishes, then the kinetic term gives us to a metric on the corresponding moduli space. By combining some part of determinant for the induced metric for M2-brane world-volume with 3-form potential, one reads off the corresponding potential. Then the potential has the factor

$$\Delta^{-\frac{3}{2}} - W_{gs} = a^{\frac{3}{2}} \left(a^2\cos^2\theta + b^2\sin^2\theta\right) \left[1 - \sqrt{1 - \frac{16\,(ab-1)\sin^2\theta\cos^2\theta}{\left(a^2\cos^2\theta + b^2\sin^2\theta\right)^2}}\,\right]$$

where we substituted (4.5), (4.4) and (4.8). This potential vanishes for $\theta = \frac{\pi}{2}$. Of course, there exists trivial solution for $\theta = 0$ where the metric becomes zero. On this subspace, the metric on the 7-dimensional moduli space transverse to the M2-branes is given by

$$ds^2|_{\text{moduli}} = \sqrt{a}\, L^2\, e^A\, d\Omega_6^2 + e^A\, a^{\frac{3}{2}}\, b^2\, dr^2 \tag{4.9}$$

where we used the fact that for $\theta = 0$ we have simple relations from (4.4) and (4.5): $\xi^2 = b^2$ and $\Delta = a^{-1} b^{-\frac{4}{3}}$.

[4] The superpotential W is the same as (2.3) $W = \frac{1}{8}\sqrt{a^3\left[(a^2+7b^2)^2 - 112\,(ab-1)\right]}$ and the superpotential by [5] is $W_{AI} = \frac{1}{16W} a^3 \left[\left(48(1-ab) + (a^2-b^2)(a^2+7b^2)\right)\cos^2\theta + 8\,(1-ab) + b^2\left(a^2+7b^2\right)\right]$.

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As we approach the IR critical point, we can introduce a new radial coordinate $u \simeq e^{\frac{1}{2}A(r)}$ with $\frac{du}{dr} = \frac{1}{L}\sqrt{\frac{6}{5}}\sqrt{a}\, b\, u$ to obtain the asymptotic form for the metric by inserting (4.2), (2.4) and the IR critical value of W in table 1 into (4.9)

$$ds^2|_{\text{moduli}} = \frac{5^{\frac{3}{4}}3^{\frac{1}{8}}}{6^{\frac{3}{4}}} L^2 \left(du^2 + \frac{6}{5} u^2\, d\Omega_6^2 \right). \tag{4.10}$$

Also at the IR critical point, one can see that

$$\frac{d}{dr}\left(e^{A(r)}\sqrt{a}\right)|_{\text{IR}} = \frac{2}{L}\sqrt{\frac{6}{5}}\, a\, b\, e^{A(r)}|_{\text{IR}}$$

by using the supersymmetric flow equations (2.4) we introduced in section 2. The mass spectrum for the $\frac{\sqrt{7}}{2}\lambda$ around G_2 fixed point was computed in [6] and it is $\frac{5}{6}\left(6+\sqrt{3}\right)$. At the IR end of the flow, $A(r) \sim \frac{2\sqrt{\frac{36\sqrt{2}\,3^{1/4}}{25\sqrt{5}}}}{L} r$ with $g \equiv \frac{\sqrt{2}}{L}$ from the solution (2.4) for $A(r)$ and $W = \sqrt{\frac{36\sqrt{2}\,3^{1/4}}{25\sqrt{5}}}$ from table 1. Moreover, $u \sim e^{\frac{\sqrt{\frac{36\sqrt{2}\,3^{1/4}}{25\sqrt{5}}}}{L} r} \sim e^{\frac{A(r)}{2}}$ above. Then S becomes $S = (\Phi_1^2 + \cdots \Phi_7^2)^{\frac{6}{5}}$ in the boundary theory. The power $\frac{6}{5}$ comes from the factor in the metric (4.10) of the moduli. Obviously, from the tensor product between **7** and **7** of G_2 representation, one gets a singlet **1** as before. For the superfield $S(x,\theta)$, the action looks like $\int d^3x d^2\theta S(x,\theta)$. The component content of this action can be worked out straightforwardly using the projection technique. This implies that the highest component field in θ-expansion, the last element in table 2, has a conformal dimension $6 + \frac{5}{6}\sqrt{3}$ in the IR as before.

We have presented the gauge invariant combinations of the massless superfields of the gauge theory whose G_2 quantum numbers exactly match the four multiplets in tables 3, 4, 5 observed in the supergravity. There exists one additional Wess-Zumino multiplet in table 2 which completes the picture.

5. Conclusions and outlook

By analyzing the mass-deformed Bagger-Lambert theory(or the mass-deformed $\mathrm{U}(2)\times\mathrm{U}(2)$ Chern-Simons gauge theory with level $k=1$ or 2), preserving G_2 symmetry, with the addition of mass term for one of the eight adjoint superfields, one identifies an $\mathcal{N}=1$ supersymmetric membrane flow in three dimensional deformed theory with the holographic $\mathcal{N}=1$ supersymmetric RG flow in four dimensions. Therefore, the $\mathcal{N}=8$ gauged supergravity critical point is indeed the holographic dual of the mass-deformed $\mathcal{N}=8$ BL theory(or the mass-deformed $\mathrm{U}(2)\times\mathrm{U}(2)$ Chern-Simons gauge theory with level $k=1$ or 2). So far, we have focused on the particular mass deformation (3.2) preserving G_2 symmetry. As we mentioned in introducation, there are three more nonsupersymmetric critical points, $\mathrm{SO}(7)^+, \mathrm{SO}(7)^-$ and $\mathrm{SU}(4)^-$. It would be interesting to find out all the possible cases for the mass deformations and see how they appear in the $AdS_4 \times \mathbf{X}^7$ background in the context of $\mathrm{U}(N)\times\mathrm{U}(N)$ Chern-Simons gauge theory.

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A. Branching rules

In this appendix we list some useful branching rules with the help of [78]. In order to obtain the branching rule SO(8) $\rightarrow G_2$, we need to consider the following two branching rules, SO(8) $\rightarrow$ SO(7) and SO(7) $\rightarrow G_2$. The former is given by

SO(8) $\rightarrow$ SO(7) branching rule:

$$\begin{aligned}
\mathbf{1} &\rightarrow \mathbf{1}, \\
\mathbf{8}_v &\rightarrow \mathbf{8}, \\
\mathbf{8}_s &\rightarrow \mathbf{1} \oplus \mathbf{7}, \\
\mathbf{8}_c &\rightarrow \mathbf{8}, \\
\mathbf{28} &\rightarrow \mathbf{7} \oplus \mathbf{21}, \\
\mathbf{35}_v &\rightarrow \mathbf{35}, \\
\mathbf{35}_s &\rightarrow \mathbf{1} \oplus \mathbf{7} \oplus \mathbf{27}, \\
\mathbf{35}_c &\rightarrow \mathbf{35}, \\
\mathbf{56}_v &\rightarrow \mathbf{8} \oplus \mathbf{48}, \\
\mathbf{56}_s &\rightarrow \mathbf{21} \oplus \mathbf{35}, \\
\mathbf{56}_c &\rightarrow \mathbf{8} \oplus \mathbf{48}
\end{aligned} \tag{A.1}$$

and the latter is given by

SO(7) $\rightarrow$ G_2 branching rule:

$$\begin{aligned}
\mathbf{1} &\rightarrow \mathbf{1}, \\
\mathbf{7} &\rightarrow \mathbf{7}, \\
\mathbf{8} &\rightarrow \mathbf{1} \oplus \mathbf{7}, \\
\mathbf{21} &\rightarrow \mathbf{7} \oplus \mathbf{14}, \\
\mathbf{27} &\rightarrow \mathbf{27} \\
\mathbf{35} &\rightarrow \mathbf{1} \oplus \mathbf{7} \oplus \mathbf{27}, \\
\mathbf{48} &\rightarrow \mathbf{7} \oplus \mathbf{14} \oplus \mathbf{27}.
\end{aligned} \tag{A.2}$$

Combining the two results (A.1) and (A.2), one obtains the following branching rule which is necessary to analyze the section 2.

SO(8) → G_2 branching rule:

$$\begin{aligned}
\mathbf{1} &\rightarrow \mathbf{1}, \\
\mathbf{8} &\rightarrow \mathbf{1} \oplus \mathbf{7}, \\
\mathbf{28} &\rightarrow \mathbf{7} \oplus \mathbf{7} \oplus \mathbf{14}, \\
\mathbf{56} &\rightarrow \mathbf{1} \oplus \mathbf{7} \oplus \mathbf{7} \oplus \mathbf{14} \oplus \mathbf{27}, \\
\mathbf{70} &\rightarrow \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{7} \oplus \mathbf{7} \oplus \mathbf{27} \oplus \mathbf{27}. \qquad \text{(A.3)}
\end{aligned}$$

It is better to present the following branching rule for the correpondence between $\mathcal{N} = 1$ and $\mathcal{N} = 2$ critical points.

G_2 → SU(3) branching rule:

$$\begin{aligned}
\mathbf{1} &\rightarrow \mathbf{1}, \\
\mathbf{7} &\rightarrow \mathbf{1} \oplus \mathbf{3} \oplus \overline{\mathbf{3}}, \\
\mathbf{14} &\rightarrow \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \mathbf{8}, \\
\mathbf{27} &\rightarrow \mathbf{1} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{6}} \oplus \mathbf{8}. \qquad \text{(A.4)}
\end{aligned}$$

B. Explicit form for $\widetilde{A}_1$-tensor

The R^8 coordinates x_I in (4.6) are related to the ones X_I in (4.3) as follows:

$$\begin{aligned}
X_1 &\equiv \frac{1}{\sqrt{2}}(x_2 - x_6), \qquad & X_2 &\equiv -\frac{1}{\sqrt{2}}(x_3 - x_7), \\
X_3 &\equiv \frac{1}{\sqrt{2}}(x_4 - x_8), \qquad & X_4 &\equiv -\frac{1}{\sqrt{2}}(x_1 - x_5), \\
X_5 &\equiv \frac{1}{\sqrt{2}}(x_2 + x_6), \qquad & X_6 &\equiv \frac{1}{\sqrt{2}}(x_3 + x_7), \\
X_7 &\equiv \frac{1}{\sqrt{2}}(x_4 + x_8), \qquad & X_8 &\equiv \frac{1}{\sqrt{2}}(x_1 + x_5) = \cos\theta
\end{aligned}$$

and we list here for the expressions for $\widetilde{A}_1$-tensor in (4.7)

$$\begin{aligned}
\widetilde{A}_1^{88} &= \frac{(1+a+i\sqrt{-1+ab})^3}{(2+a+b)^{\frac{7}{2}}}\Big[8 + 16b - 8ab + 12b^2 - 12ab^2 + a^2b^2 + 4b^3 - 6ab^3 \\
&\quad + b^4 + (a-b)(2+a+b)(8+a^2+a(2-6b)+b(2+b))\cos^2\theta + \sqrt{-1+ab} \\
&\quad \times(-4i(1+b)(2+b(2-a+b)) + 4i(2+a+b)(2+a+a^2+b-2ab+b^2)\cos^2\theta)\Big], \\
\widetilde{A}_1^{mm} &= \frac{1}{\sqrt{2+a+b}}\Big[a(1+a)(b^2+(a-b)(a+b)\cos^2\theta) - 4(2+b)(-1+ab)X_m^2 \\
&\quad -i\sqrt{-1+ab}(a(b^2+(a-b)(a+b)\cos^2\theta) + 4(-2+(-1+a)b)X_m^2)\Big], \\
\widetilde{A}_1^{mn} &= \frac{4}{\sqrt{2+a+b}}\Big[-i(-2+(-1+a)b)\sqrt{-1+ab} - (2+b)(-1+ab)\Big]X_m X_n, \quad m \neq n \\
\widetilde{A}_1^{m8} &= \frac{4(1+a+i\sqrt{-1+ab})^3}{(2+a+b)^{\frac{3}{2}}(a-b+2i\sqrt{-1+ab})}\Big[2 - 2ab + i(a-b)\sqrt{-1+ab}\Big]X_m\cos\theta, \qquad \text{(B.1)}
\end{aligned}$$

where $m, n = 1, 2, \cdots, 7$. Also one can obtain the full expressions for the $\widetilde{A}_2$-tensor of the theory which we do not present here. We have checked from these $\widetilde{A}_1$ and $\widetilde{A}_2$-tensors that the scalar potential has a simple form and it is given by $V = 2a^3 \left[a^2 + b^2 + (a^2 - b^2)\cos 2\theta\right]^2$.

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